B Time Manipulation

B.1. Consider a function of time $x(t)$.

(a) **Time shifting**: $t \to t - T$

(i) When $T > 0$, $x(t-T)$ is $x(t)$ right-shifted (**delayed**) by T.

(ii) When $T < 0$, $x(t-T)$ is $x(t)$ left-shifted (**advanced**) by $|T|$.

(b) **Time scaling** (horizontal scaling): $t \to at$

(i) When $0 < a < 1$, $x(at)$ is $x(t)$ **expanded** in time (horizontally) by a factor of $\frac{1}{a}$.

(ii) When $a > 1$, $x(at)$ is $x(t)$ **compressed** in time (horizontally) by a factor of a.

- Note that the signal remains anchors at $t = 0$. In other words, the signal at $t = 0$ remains unchanged.

(c) **Time inversion** (or folding):

- $x(t)$ is the mirror image of $x(t)$ about the vertical axis.

Example B.2. A function of the form $x(mt + c)$ can be viewed as

(a) $x((mt) - (-c))$: First right-shift $x(t)$ by $-c$. Then scale horizontally by a factor of $\frac{1}{m}$.

$x(t) \xrightarrow[t \to t+c]{} x(t+c) \xrightarrow[t \to mt]{} x(mt+c)$

(b) $x\left(m\left(t - \left(-\frac{c}{m}\right)\right)\right)$: First scale $x(t)$ horizontally by a factor of $\frac{1}{m}$. Then, right-shift by $-\frac{c}{m}$.

$x(t) \xrightarrow[t \to mt]{} x(mt) \xrightarrow[t \to t+\frac{c}{m}]{} x(m(t+\frac{c}{m})) = x(mt+c)$

These two approaches are illustrated in Figure 87.

Alternatively, it may be easier to look at where the key points in the plot will show up in the new plot. For example, let's consider the leftmost point in the original plot of $x(t)$. Note that it occurs when the argument of $x(t)$ is a. So, in the plot of $x(mt + c)$, it will occur at t such that $mt + c = a$. Therefore, it will be at $t = \frac{a-c}{m}$.

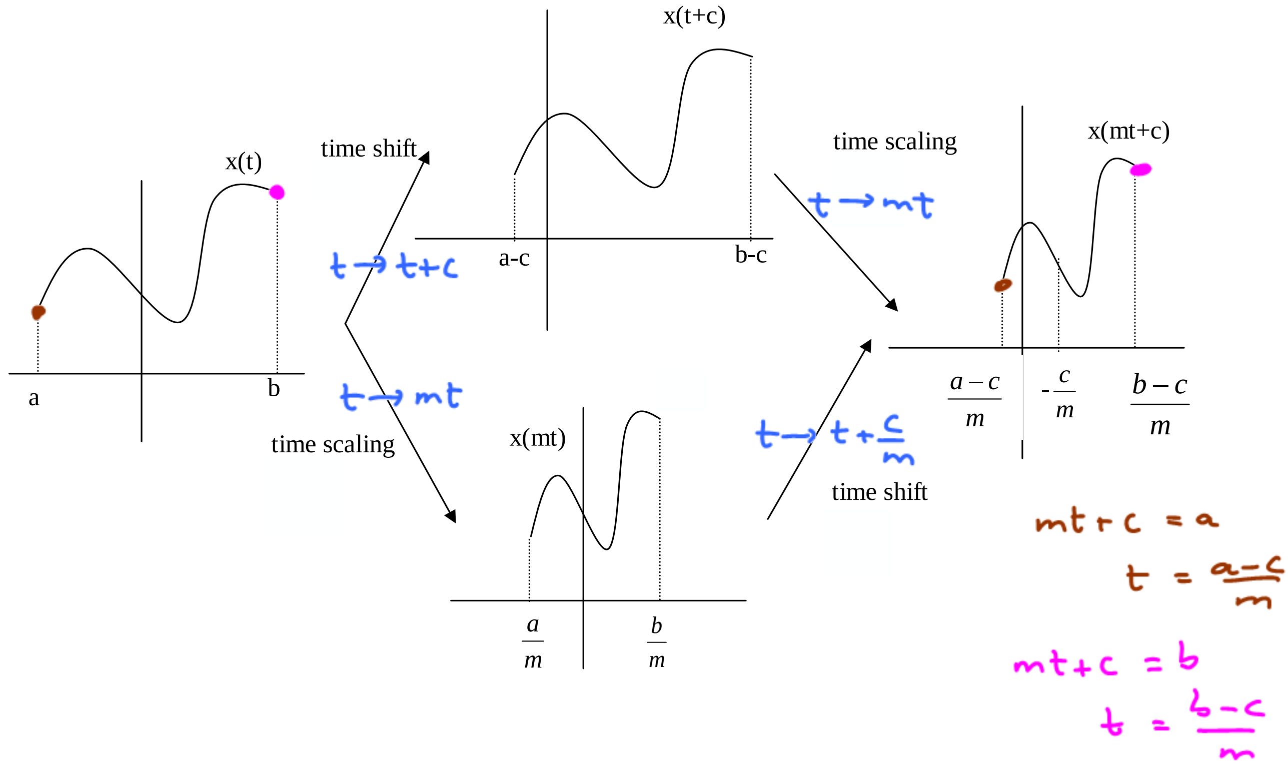

Figure 87: Two approaches for drawing $x\,(mt + c)$.

Example B.3. Consider the function $x(t)$ given in Figure 88.

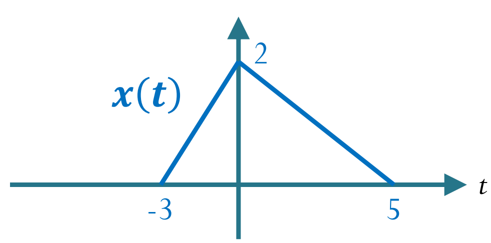

Figure 88: The function $x(t)$ used in Example B.3.

(a) Find the area under $x(t)$.
Solution: See the top part of Figure 89.

(b) Plot and find the area under the new function $x(-t)$.
Solution: See Figure 89.

(c) Plot and find the area under $x(2t)$.
Solution: See Figure 90 for derivation of the plot then see Figure 91 for calculation of the area.

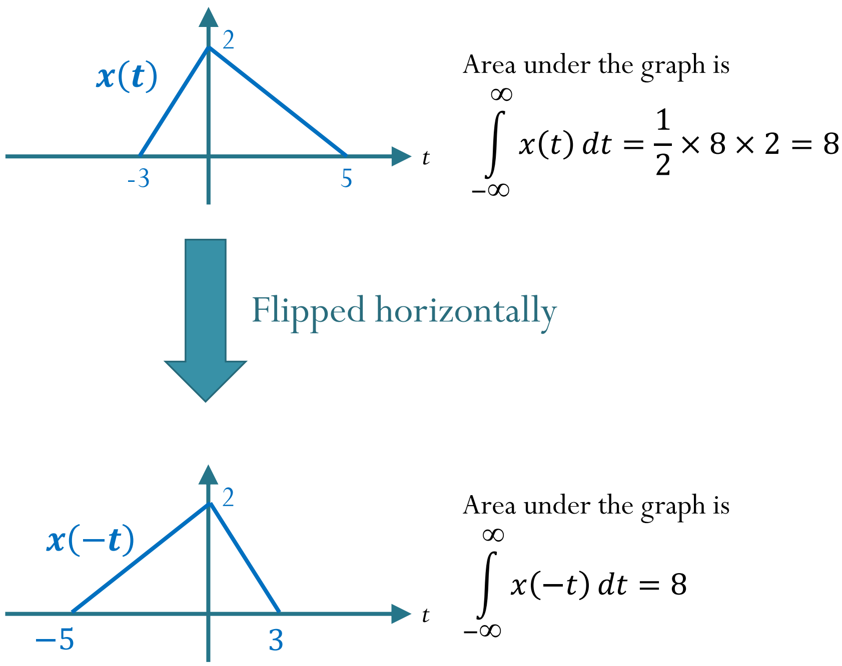

Figure 89: Example of time inversion.

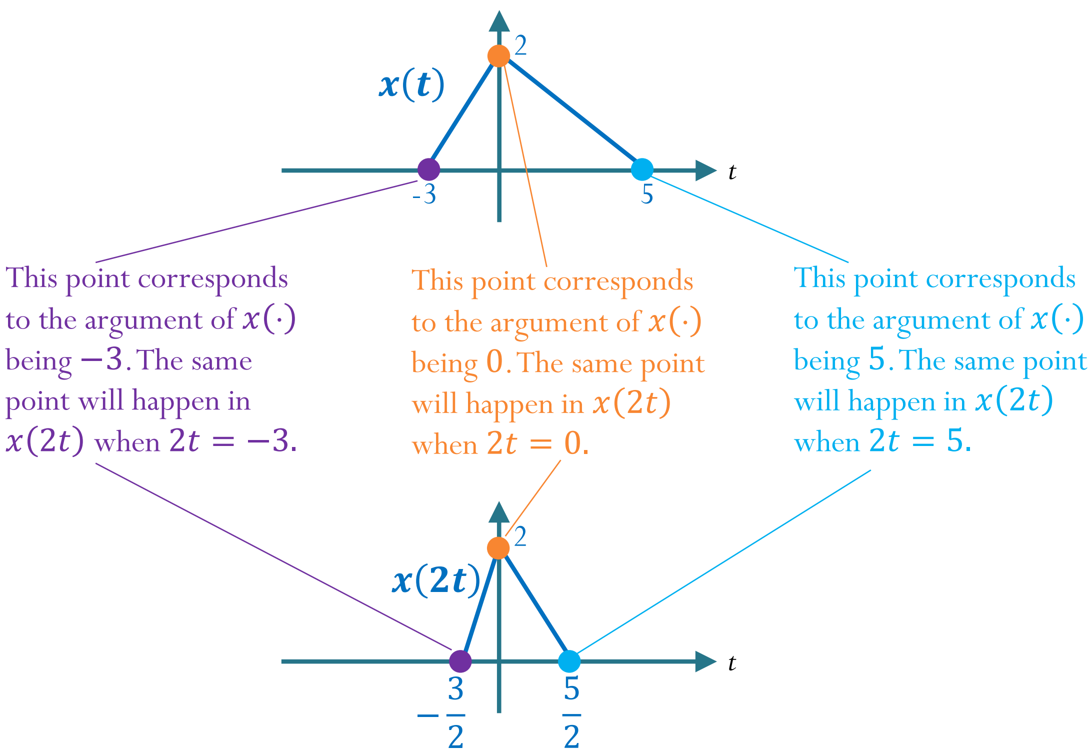

Figure 90: Example of time scaling.

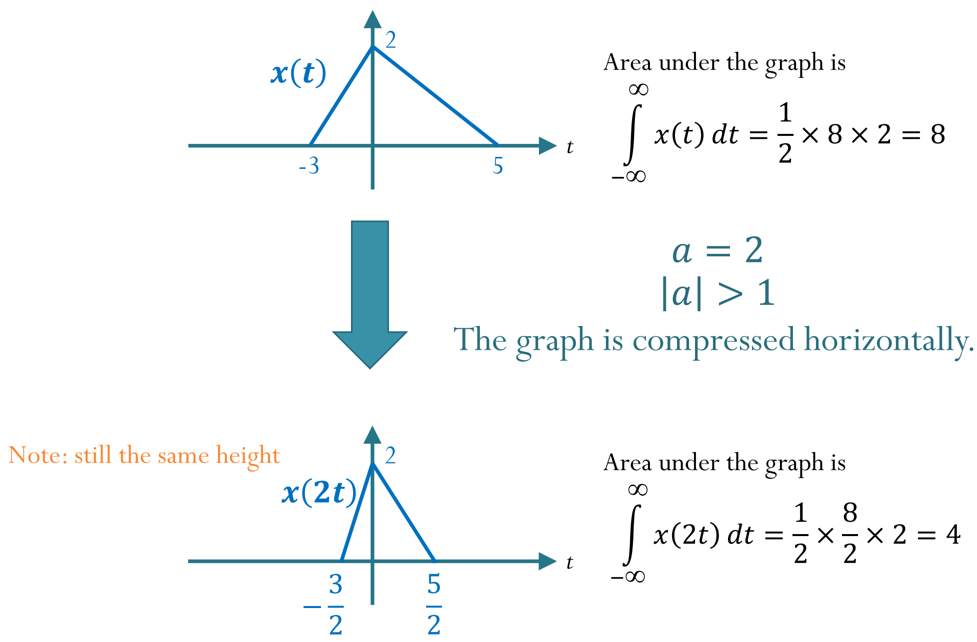

Figure 91: Calculating the area when the function is scaled horizontally.

B.4. Figure 92 shows the delta functions as limits of rectangular functions whose width are compressed to 0 while the areas are kept constant.

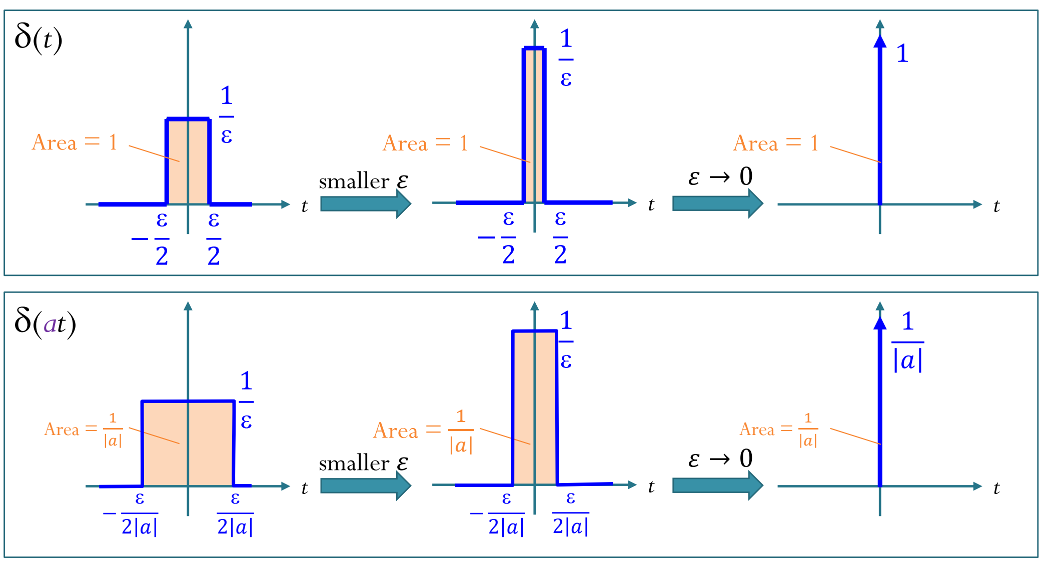

Figure 92: Delta functions as limits of rectangular functions.Here, $a = \frac{1}{2}$.